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Effect of Internal Radial Clearance on Performance of Bearing

***Abstract* : The rolling bearings dynamic behavior analysis is a critical condition to determine the machine vibration response. The rolling bearing, with outer ring fixed, is a multi body mechanical system with rolling elements that transmit motion and load from the inner raceway to the outer raceway. The specific construction of a bearing has a decisive influence on its dynamic behavior. The paper defines a vibration model of a rigid rotor supported by rolling element bearings. By application of the defined model, the parametric analysis of the influence of internal radial clearance value and number of rolling elements influence on the rigid rotor vibrations in unloaded rolling element bearing was presented.**

***Index Terms* : analytical model, rolling bearing, ball passage frequency, internal radial clearance.**

I. INTRODUCTION

Ball bearings are among the most important and frequently used components of the machines; however bearings may contain manufacturing errors or mounting defects. Such errors cause vibration, noise, and even failure of the whole system, which leads to expensive claims for damage. To avoid this and to ensure rapid and cheap production there is a need for a quick end-test of the machines to determine any bearing faults. The first step in bearing-fault detection during run-up would be a numerical model for the bearing-vibration response due to faults during the run-up. A well-defined vibration signal during the run-up of a faulty bearing could be used to find a suitable method for the fault diagnostic. A lot of research work has been done to model the vibration response of a bearing due to faults at a constant rotational speed.

Tadina [1] presented an improved 2D bearing model for investigation of the vibrations of a ball-bearing during run-up. They presented numerical model assumes deformable outer race, which is modeled with finite elements, centrifugal load effects and radial clearance.

Various surface defects due to local deformations are introduced into the developed model. The detailed geometry of the local defects is modeled as an impressed ellipsoid on the races and as a attended sphere for the rolling balls. The obtained equations of motion were solved numerically with a modified New mark time-integration method for the increasing rotational frequency of the shaft. The simulated vibration response of the bearing with different local faults was used to test the suitability of the continuous wavelet transformation for the bearing fault identification and classification. The experimental validation of the simulation was done by wavelet transformation.

Upadhyay [2] presents a mathematical model to investigate the nonlinear dynamic behavior of a high speed rotor-bearing system due to defects of rolling elements. They used two defects for the study which are off size rolling element and waviness of rolling element. In the formulation, the contacts between rolling elements and inner/outer races are considered as nonlinear springs and also used nonlinear damping, which is developed by correlating the contact damping force with the equivalent contact stiffness and contact deformation rate. The equations of motion are formulated using Lagrange's equation, considering the vibration characteristics of the individual components such as inner race, outer race, rolling elements and rotor. They solved the equation of motion using numerical line integration technique.

Rubio [3] developed rolling bearing analytical formulation, the contact between rolling element and raceways is considered as nonlinear springs and their stiffness are obtained by using Hertzian elastic contact deformation theory. Equation of motion is developed using Lagrange's formulation and simulation is validated using Algor© code and finite element method in time domain with dynamic events.

Wensing [4] considered the stiffness and damping of the elastohydro dynamic lubricated (EHL) contact between ball and guiding rings. They formulated the equation using finite element modeling and time integration and component mode synthesis (CMS) is used as method of solution of equation. Experimental validation is done by vibration test spindles.

Tillema [5] presented the 3D nonlinear time dependent computational model of rolling element bearing.

They studied the vibration generation and vibration transmission characteristics of the rolling element. The formulation was done by Maxwell model of time domain equation. The equation of motion was solved by Newmark time integration method and Newton-Rapson method. The mathematical simulation was validated by experiments on a rotor dynamic test rig.

Jang [6] used the two type of defect of the bearing to study the dynamic behavior of the bearing. Those were wavieness of ball and waviness of the races. Problem formulation was done by Runge-Kutta-Fehlberg algorithm and validation was done with those of prior researchers.

Purohit [7] developed the nonlinear dynamic model of ball bearing with effect of varying preload and number of balls. The mathematical model was solved by Newmark-β with Newton-Rapson method. The results are validated using FFT.

Harsha [8] presented nonlinear dynamic analysis of a rail axle-bearing system due to the effects of the number of rolling elements has been studied. The result showed The results show the appearance of instability and chaos in the dynamic response as the speed of the axle-bearing system is changed. Period doubling and mechanism of intermittency have been observed which lead to chaos. The appearance of regions of periodic, sub-harmonic and chaotic behavior is seen to be strongly dependent on the radial clearance. Poincaré maps, phase plots and frequency spectra are used to elucidate and to illustrate the diversity of the system behavior.

II. DEFECTS CONSIDER FOR STUDY

The dynamic performance of bearing is highly influential on performance of any machine. More specifically, the presence of bearing defects often results in reduced efficiency, or even severe damage of the machine under consideration. System rigidity in machine tool is extremely important because the magnitude of deflection under load determines machining accuracy. The bearing stiffness is main factor that influence system rigidity. Therefore, bearing preload is needed to enhance system rigidity and to increase running accuracy. Bearing preload can be regarded as negative internal clearance. A proper amount of negative bearing clearance is desirable in order to stiffen the support of the spindle. However, inappropriate negative bearing clearance can cause excessive rolling contact stresses and eventually lead to bearing seizure. Therefore, Proper internal clearance have to be selected in order to prevent bearing seizure and to improve bearing stiffness.

III. THE EQUATION OF MOTION

The complex motion of balanced rigid rotor sleeve in rolling element bearings and found that the trajectory of rotor cross-section center in bearings with the internal radial clearance and idealized geometry could be represented by the equation of absolute sinusoid:

$$y = \Delta \left| \sin \frac{\pi s}{\lambda} \right| \qquad (1)$$

Where Δ is the peak-to-peak (pp) rotor vibration amplitude, λ the arc distance between the bearing rolling elements and s the curviliniear coordinate of bearing rolling elements brunt.

The path of rolling elements brunt is equal to pitch diameter of the cage, ie, centers of rolling elements are moving on the cage pitch circle. Based on this , the arc distance between the rolling bearing elements (λ) can be obtained according to the following equation:

$$\lambda = \frac{\pi d_c}{z} \qquad (2)$$

Also, the curvilinear coordinate of bearing rolling elements brunt (s) can be expressed as

$$s = r_c \varphi = \frac{1}{2} d_c \varphi \qquad (3)$$

If the expression (3) and the expression (2), are applied to (1), after rearrangement:

$$y = \Delta \left| \sin \frac{z}{2} \varphi \right| \qquad (4)$$

Angular coordinate of rolling elements brunt (φ) can be expressed in function of time over cage frequency ω_c according to the following equation:

$$\varphi = \omega_c t \qquad (5)$$

But $\omega_c = \frac{\omega}{2}\left(1 - \frac{d_b}{d_c}\cos\alpha\right)$, so

$$\varphi = \omega_c t = \frac{\omega}{2}\left(1 - \frac{d_b}{d_c}\cos\alpha\right) t \qquad (6)$$

Where ω is shaft frequency, d_b the diameter of rolling elements, α the bearing contact angle and t the time.

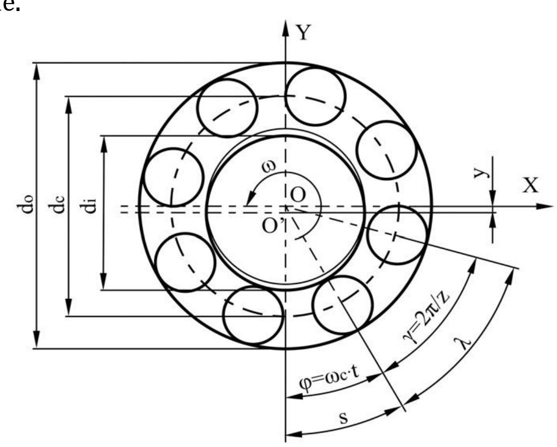

Fig. 1. Schematic view of rolling element bearings kinematics.

Now the equation for curvilinear coordinated s can be given as

$$s = \frac{1}{2}\omega_c d_c t = \frac{\omega}{2}\left(1 - \frac{d_b}{d_c}\cos\alpha\right)\frac{d_c t}{2} \qquad (7)$$

Furthermore, if the terms for s and λ are added to eq. (1)one can obtained:

$$y = \Delta\left|\sin\left(\frac{z}{2}\omega_c t\right)\right| = \Delta\left|\sin\left[\frac{z}{2}\frac{\omega}{2}\left(1 - \frac{d_b}{d_c}\cos\alpha\right)t\right]\right| \qquad (8)$$

Or

$$y = \Delta\left|\sin\left(\frac{f_{bp}}{2}t\right)\right| \qquad (9)$$

Where

$$f_{bp} = \frac{z}{2}\omega\left(1 - \frac{d_b}{d_c}\cos\alpha\right) \qquad (10)$$

In above equationf_{bp}, as previously noted, represent the ball passage frequency (BPF).
According to this, the rotor cross-section centre in relation to the centre opening in the bearing housing vibrates constantly during bearing operation. These vibrations can be represented by the equation of absolute sinusoid (9).The frequency of these vibrations is equal to ball passage frequency, and its amplitude is equal to rotor vibrations pp-amplitude. The pp-amplitude (Δ) is equal to the difference of maximum divergence from a reference rotor position on one and the other side

$$\Delta = y_{max} - y_{min} \qquad (11)$$

IV. THEORETICAL ANALYSIS OF RIGID ROTOR VIBRATIONS AMPLITUDE CHANGE IN UNLOADED ROLLING ELEMENT BEARINGS

In an unloaded rolling element bearing ,with internal radial clearance, the inner race is supported by two rolling elements as shown in Fig. 2. This support case often occurs in bearings subjected to low values of external radial load, in which the contact deformations are not sufficient enough to enable the participation of other rolling elements in the transfer of load. In such cases, the bearing structure can be observed as a system of rigid bodies ,in which the transfer of load from the inner to the outer race goes over one or two rolling elements. In a system of rigid bodies , all links are geometric and all movements can be analyzed based on geometrical laws. The rotor centre will vibrate between two extreme positions, which are determined by position of the set of rolling elements, as is shown in Fig. 2. As shown in Fig. 2a, sleeve centre will be moved in relation to the outerrace centre for half the value of internal radial clearance (e). This is the minimum deviation of the rotor in relation to the opening centre in the bearing case, i.e.:

$$y_{min} = \frac{e}{2} = r_o - r_i - d_b \qquad (12)$$

The maximum deviation of the rotor in relation to the centre of the inner race occurs in a case of symmetrically distributed rolling elements in relation to the bearing vertical axis (Fig. 2b). The maximum value of rotor divergence y_{max}, can be obtained based on the scheme shown in Fig. 3[28].
From Fig. 3 one can see that the following equality applies:

$$\overline{B_0A} = \sqrt{\overline{O'B_o}^2 - \overline{O'A}^2} = \sqrt{\left(\frac{d_i - d_b}{2}\right)^2 - y_{max}^2\sin^2\frac{\gamma}{2}} \qquad (13)$$

Article $y_{max}^2\sin^2\frac{\gamma}{2}$ can be ignored, because y_{max}^2 is the small dimension of higher order in relation to the d_i and d_b, so

$$\overline{B_oA} = \frac{d_i - d_b}{2} \qquad (14)$$

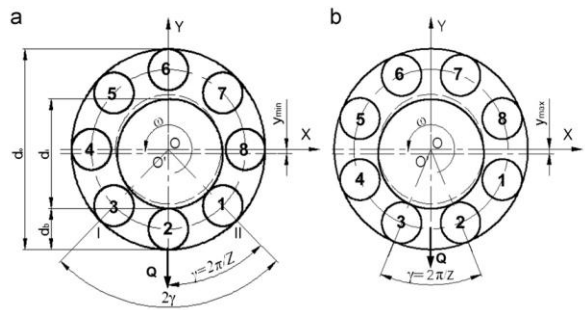

Fig 2. The extreme rotor positions in the case of support by two rolling elements.

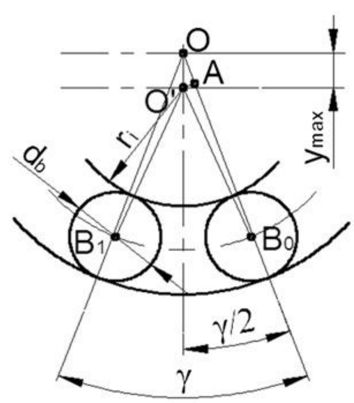

Fig.3. Scheme of the maximum divergence of the rotor leaned on a two rolling bodies.

If this is inserted in the equation $\overline{OB_o} = \overline{OA} + \overline{B_oA}$:

$$\frac{d_i - d_b + e}{2} = y_{max}\cos\frac{\gamma}{2} + \frac{d_i + d_b}{2} \qquad (15)$$

After systemization the maximum deviation of the rotor y_{max}:

$$y_{max} = \frac{e}{2}\frac{1}{\cos\left(\gamma/_{2}\right)} \tag{16}$$

The difference of the extreme values of rotor divergence gives the maximum gap ,i.e .pp-amplitude (Δ_1) of rigid rotor centre vibrations in the case of support by two rolling elements, according to

$$\Delta_1 = y_{max} - y_{min} = \frac{e}{2}\left(\frac{1}{\cos\frac{\gamma}{2}} - 1\right) \tag{17}$$

By analyzing expression (17) one can see that the pp-amplitude of rigid rotor vibrations in rolling element bearings, in unloaded bearing, depends on the size of the internal radial clearance and angle distance between the rolling elements. Since the angle between rolling elements, is directly dependent on their number, one can say that the rotor vibration pp-amplitude, is directly determined by the value of the internal radial clearance and the total number of rolling elements in the bearing. Other bearing characteristics do not have influence on the size of pp-amplitude. If the external load is relatively low, bearings with the same number of rolling elements and the same value of internal radial clearance will have the same value of vibration, regardless of the size and dimensions of the bearings structure elements.

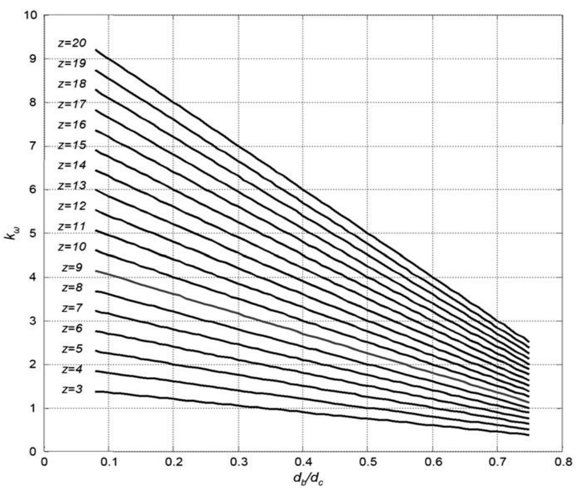

Fig. 4. Dependence of k_ω on $d_b/\ d_c$ ratio and number of rolling elements (z) for single row radial contact ball bearing.

V. PARAMETRIC ANALYSIS OF THE INFLUENCE OF BEARING CONSTRUCTION ON BALANCED RIGID ROTOR VIBRATION CHARACTERISTICS

A. Vibration Frequency

Based on previous analysis it can be concluded that vibration frequency of rigid rotor in idealistic rolling element bearing, with internal radial clearance, is equal to ball passage frequency. The frequency change is given by Eq. (10) and there from it is clear that ball passage frequency depends directly on rotor frequency and bearing construction (number of rolling elements z, ratio of rolling element diameter db and cage diameter dc and contact angle a). Furthermore, from Eq. (10) it is obvious that BPF does not depend on the value of internal radial clearance. If, in Eq.(10),use that

$$k_\omega = \frac{z}{2}\left(1 - \frac{d_b}{d_c}\right)\cos\alpha \tag{18}$$

it can be seen that this frequency is linearly dependable on the shaft frequency, with linearity coefficient k_ω

$$f_o = k_\omega\,\omega \tag{19}$$

Coefficient k_ω represent the linearity coefficient whose value is a function of bearing construction. This coefficient is a ratio of BPF and shaft frequency and shows how much BPF is larger than shaft frequency. k_ω is a function of total number of rolling elements (z), ratio of rolling elements diameter and cage diameter (db/dc), as well as contact angle (α). By multiplication of k_ω with shaft frequency, it is easy to obtain the BPF. Fig. 4 shows the dependence of coefficient k_ω on the bearing construction for radial contact ball bearing. The diagram enables the obtaining of k_ω value in respect to ratio db/dc, for different values of total number of rolling elements (z). By analyzing Fig. 4 it is clear that k_ω linearly decreases with increase of ratio db/dc, while its value increases with the increase of total number of rolling elements. Based on bearing construction (number of rolling elements and db/dc ratio), the BPF can be as much as ten times larger than the shaft frequency. Fig. 5 gives the dependence of coefficientk_ω, thus BPF, on the total number of rolling elements in radial contact rolling bearing. Based on Fig. 5 and Eq. (10) it can be concluded that BPF is linear function of total number of rolling elements, with linearity coefficient that depends on ratio db/dc and shaft frequency. Fig. 5 also shows that with the increase of number of rolling elements, k_ω also increases and thus BPF. Steeper lines have lower values of (db/dc), i.e .the increase of number of rolling elements with smaller values of db/dc has greater influence on increase of BPF.

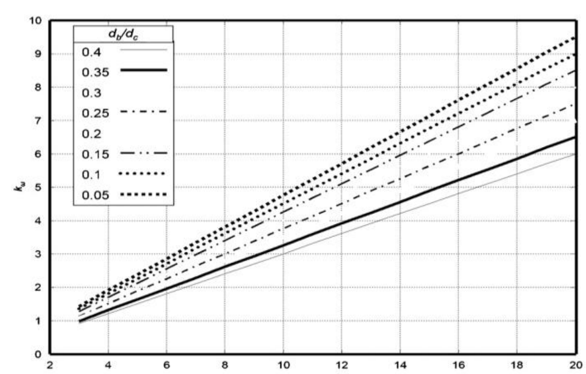

Fig. 5. Dependence of k_ω on the number of rolling elements for different values of ratio between rolling element diameter and cage pitch diameter (db/dc).

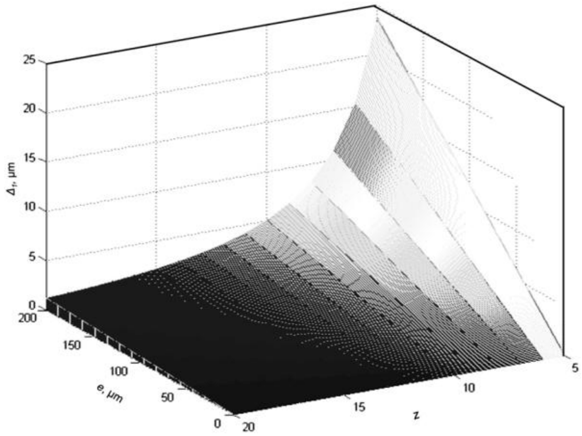

Fig. 6. Three-dimensional diagram of BPF dependency in relation to value of internal radial clearance and total number of rolling elements in the case of reclining on two rolling elements.

B. Vibrations Amplitude

The influence of bearing construction on BPV amplitude, in unloaded bearing, can be analyzed by Eq. (17) which defines the pp-amplitude of balanced rigid rotor centre vibrations in a case of support by

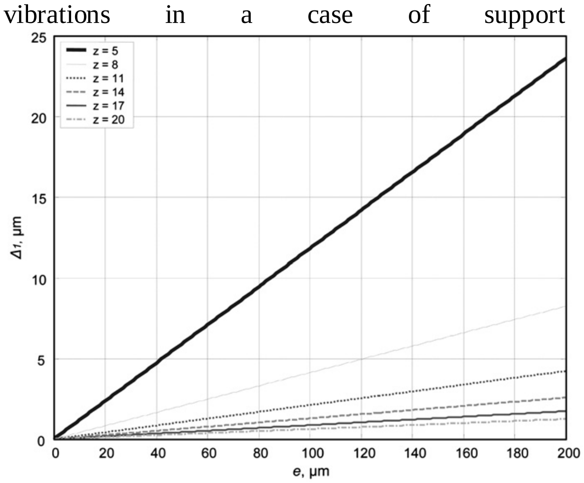

Fig.7. Balanced rigid rotor vibration amplitude dependency in relation to value of internal radial clearance in the case of support on two rolling elements

two rolling elements. According to (17), amplitude linearly depends on the value of internal radial clearance, with linearity coefficient that is a function of angle between rolling elements (γ):

$$a = tg(\alpha) = \frac{1}{2}\left(\frac{1}{\cos\gamma/2} - 1\right) \quad (20)$$

where a is a linearity coefficient. The total number of elements in the bearing determines the angle between rolling elements (γ), so the linearity coefficient is direct function of the total number of rolling elements. Therefore, the analysis of the internal radial clearance impact on BPV amplitude is best to perform for different values of total number of rolling elements. Fig. 6 gives the three-dimensional dependency of pp- amplitude in relation to value of internal radial clearance and total number of rolling elements. Fig. 7 gives the dependency of pp-amplitude on the internal radial clearance for different values of total number of rolling elements. With the increase of value of internal radial clearance ,the value of pp-amplitude also linearly increases. The increment is bigger with the decrease of total number of rolling elements. From Fig. 7 it is clear that with the increase of total number of rolling elements the dependency lines tend towards horizontal, i.e. the angle that they precede in relation to horizontal axis decreases.

VI. CONCLUSIONS

Based on the numerical analysis it can be concluded that:

1. The centre of rotor cross-section continuously vibrates in regard to bearing housing opening. These vibrations can be represented by the equation of the absolute sinusoid. By this equation, the vibration frequency of the rigid rotor in the ideal rolling element bearing with internal radial clearance is equal to ball passage frequency (BPF). The amplitude of these vibrations is equal to rotor vibrations pp-amplitude and it is equal to the difference of maximum divergence from a reference rotor position in one and the other side.
2. The BPF is in linear relation to the shaft frequency, with the coefficient of linearityk_ω. Coefficient k_ω shows the impact of the rolling element bearing construction parameters on the BPF. This coefficient depends on the following: number of rolling elements (z), ratio of rolling elements diameter and the cage pitch diameter (db/dc), as well as the contact angle of the bearing (α). The value of internal radial clearance does not influence k_ω
3. The BPF linearly increases with the increase of number of rolling elements. This increase is more pronounced with the lower values of ratio between diameter of rolling elements and cage pitch diameter (db/dc).
4. The vibration amplitude on BPF, in unloaded rolling element bearing ,is linearly dependable on the value of internal radial clearance, with linearity coefficient, which is influenced by angular distance between rolling elements, i.e .the total number of rolling elements in the bearing. With the increase of the internal radial clearance, the value of amplitude increases linearly. The increase gets much bigger as the total number of rolling elements decreases.

NOMENCLATURE

y trajectory of rotor cross-section centre
Δ peak-to-peak (pp) rotor vibration amplitude
λ arc distance between the bearing rolling elements
s curvilinear coordinate of bearing rolling elements brunt
φ angular coordinate of rolling elementsbrunt
z total number of rolling elements

t time
ω shaft frequency
ω_c cage frequency
α contact angle
f_{bp} ball passage frequency (BPF)
f_{bpr} calculated values of BPF
f_{bpe} experimentally obtained values of BPF
γ angle between rolling elements
e internal radial clearance
d nominal diameter (shaft diameter)
di bearing inner raceway diameter
dc cage pitch diameter
db diameter of rolling elements
do bearing outer raceway diameter
D bearing outside diameter
ri inner raceway groove curvature radius
ro outer raceway groove curvature radius
kω coefficient a coefficient
a coefficient

REFERENCES

[1] Matej Tadina and Miha Boltezar, 2011, "Improved 2D model of a ball bearing for the simulation of vibrations due to faults during run-up," 9th International Conference on Damage Assessment of Structures (DAMAS 2011) Journal of Physics: Conference Series **305**.

[2] S. H. Upadhyay, S. P. Harsha and S. C. Jain, "Nonlinear Vibration Signature Analysis of High Speed Rotor Due to Defects of Rolling Element," Adv. Theor. Appl. Mech., Vol. 1, 2008, no. 7, 301 – 314

[3] H. Rubio, J. C. Garcia Prada, C. Catejon and E. Laniado, "Dynamic analysis of rolling bearing system using Lagrangian model Vs. FEM code," 12th IFToMM World Congress, Besançon (France), June18-21, 2007.

[4] Wensing and Jeroen Anton, "On the dynamics of ball bearings," PhD thesis, University of Twente, Enschede, The Netherlands, ISBN 90-36512298, December 1998.

[5] Hedzer Tillema and Jeroen Wensing, "Validation of rotor dynamics model with viscoelastic bearing support,"

[6] G. H. Jang and S. W. Jeong, "Nonlinear Excitation Model of Ball Bearing Waviness in a Rigid Rotor Supported by Two or More Ball Bearings Considering Five Degrees of Freedom," *ASME Trans.*, vol. 124, pp. 82–90, Jan. 2002.

[7] R. K. Purohit and K. Purohit, "Dynamic Analysis Of Ball Bearings With Effect Of Preload And Number Of Balls," *Int. J. of Applied Mechanics and Engineering, 2006, vol.11, No.1, pp.77-91*

[8] S. P. Harsha, V. H. Saran, V. K. Goel and Mats Berg, "Nonlinear Vibration Signature Analysis of Rail Axle - Bearing Systems," 2008.

[9] M. Tiwari And K. Gupta, "Dynamic Response Of An Unbalanced RotorSupported On Ball Bearings," *Journal of Sound and vibration,vol.* 238(5), pp. 757-779, 2000.